21. A. M. Kozlovtsev, V. P. Muzychenko, and A. M. Bragov, "A theoretical–experimental method for the determination of the viscoelastic characteristics of polymeric composite materials," in: Abstracts of Lectures of the 5th All-Union Conference on Composite Materials [in Russian], No. 2, Moscow State Univ. (1981).

DYNAMICS OF THE DEFORMATION OF A SPHERICAL PORE IN A PLASTIC MATERIAL

A. V. Attetkov, V. V. Selivanov, and V. S. Solov'ev

UDC 662.215.5+539.374.1

Localized high-temperature regions ("hot spots") exert a decisive effect on the stimulation of chemical decomposition in high-density heterogeneous explosives. It follows from the general laws of the dynamics of deformation of porous media that their deformation characteristics are determined by the competition of two distortion mechanisms: the production of cracks (or slip bands), and pore deformation (collapse) [1]. Consequently, the formation of localized hot spots during shock triggering of heterogeneous explosives may involve pore deformation, shear fracture, and the slip ratio of the particles of the explosive. The present article is devoted to a consideration of the general laws of pore deformation in a plastic material, and the heating of its surface during its collapse. The fundamental equation of a porous material which is being compressed was proposed in [2]. A porous material was regarded as an ideal homogeneous continuum with an additional kinematic variable α, defined as the ratio of the specific volume of the porous material to the specific volume of the matrix material. A similar approach was used in [3] in investigating the deformation of granular materials. This permits the treatment of the deformation of a porous material within the framework of a continuous medium model. Theoretical papers [4, 5] have been devoted to a description of the behavior of the parameter α. The problem reduces to the consideration of the collapse of a hollow sphere under the action of pressure applied to the outer surface, where the ratio of the inside and outside radii determines the porosity of the given material. The results obtained showed that the compressibility of the material and the shear modulus G determining the elastic and elastic–plastic phases of the pore collapse do not have a significant effect. A change of porosity occurs when all the material goes over into a state of plastic flow. Thus, for load intensities an order of magnitude larger than the yield strength Y of the material, it is accurate enough to employ a rigid-plastic model of the medium.

Let us consider the deformation of a pore in a plastic material under the action of a pressure p uniformly distributed over the outer surface. Let a_0 and a be, respectively, the initial and present values of the pore radius, b_0 and b the initial and present values of the radius of the sphere. We characterize the initial porosity m_0 of the material by the ratio of the specific volume of the pores to the volume of the continuous material. Then

$$m_0 = (a_0/b_0)^3. \quad (1)$$

We assume that the material is homogeneous, isotropic, and incompressible, with a density ρ, and that it satisfies the Mises–Hencky or the Tresca–St. Venant yield condition with a constant yield strength Y.

Using the assumption of the incompressibility of the material (ρ = const), we determine the integral of the equation of continuity

$$\partial\rho/\partial t + \partial(\rho v)/\partial r + 2\rho v/r = 0$$

in the form

$$v = \dot{a}(a/r)^2, \quad (2)$$

where r is an Eulerian coordinate; v, radial velocity; and $\dot{a} = da/dt$, velocity of the pore boundary.

Substituting the last expression and the derivatives $\partial v/\partial t$ and $\partial v/\partial r$ into Euler's equation for the case of central symmetry

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 127-131, January-February, 1983. Original article submitted December 23, 1981.

0021-8944/83/2401-0111$07.50

$$\rho(\partial v/\partial t + v\partial v/\partial r) = \partial\sigma_r/\partial r + (2/r)(\sigma_r - \sigma_\theta),$$

and using the plasticity condition in the compression conditions

$$\sigma_r - \sigma_\theta = Y, \tag{3}$$

we obtain

$$\partial\sigma_r/\partial r = -2Y/r + \rho((a^2\ddot{a} + 2a\dot{a}^2)/r^2 - 2a^4\dot{a}^2/r^5). \tag{4}$$

Here σ_r and $\sigma_\theta = \sigma_\varphi$ are the principal stresses, and $\ddot{a} = d^2a/dt^2$ is the acceleration of the pore boundary.

The stresses in the range $a \leq r \leq b$ satisfy the following boundary conditions:

$$\text{at} \quad r = a \ \sigma_r = -p_g, \quad \text{at} \quad r = b \ \sigma_r = -p. \tag{5}$$

We take the law of pressure variation in the adiabatic compression of the gas in the pore in the form

$$p_g = p_{g0}(a_0/a)^{3\gamma}, \tag{6}$$

where p_{g_0} is the initial pressure of the gas in the pore, and γ is the polytropic exponent.

The integration of Eq. (4), using the boundary conditions (5) and the incompressibility condition

$$b^3 - a^3 = b_0^3 - a_0^3, \tag{7}$$

leads to a second-order nonlinear differential equation for a:

$$\ddot{a} = A\dot{a}^2 + B, \tag{8}$$

where

$$A = \frac{1}{a_0 x}\left[\frac{(1+z)(1+z^2)}{2} - 2\right]; \quad B = \frac{p}{a_0\rho x(1-z)}\left[-\frac{2Y}{p}\ln z + \frac{p_{g0}}{p}x^{-3\gamma} - 1\right]; \quad z = \frac{a}{b} = \left[1 + x^{-3}\left(\frac{1}{m_0} - 1\right)\right]^{-1/3}; \quad x = \frac{a}{a_0}.$$

Since at time t = 0, when $a = a_0$ (x = 1), the velocity of the pore boundary is $\dot{a}(0) = 0$, and the acceleration is $\ddot{a}(0) \leq 0$, we obtain the condition

$$p \geqslant p_T = -(2/3)Y\ln(m_0) + p_{g0},$$

where p_T is the effective yield strength of the plastic material under hydrostatic pressure. This condition for $p_{g_0} = 0$ follows also from the solution of the elastic–plastic problem [4, 5]. For Y = 0 Eq. (8) describes the deformation (collapse) of a pore in an ideal incompressible liquid.

It should be noted that Eq. (8) is valid only during a compression, since in considering the expansion of a pore it is necessary to take account of the elastic properties of the material [5].

The problem of the collapse of a pore in a plastic material under the action of a constant pressure can be solved analytically by using the law of conservation of energy, if the pore radius a is taken as the argument. A similar approach was used in [6] to study the behavior of a rigid-plastic cylindrical shell under the action of internal pressure.

We write the law of conservation of energy in the form

$$\Phi_0 + E_0 = \Phi + E + W + E_\Phi,$$

or, relative to Φ_0, the initial work done by the external forces,

$$\tilde{\Phi} + \tilde{E} - \tilde{E}_0 + \tilde{W} + \tilde{E}_\Phi = 1. \tag{9}$$

or, relative to Φ is the present value of the work done by the external forces, E_0 and E are respectively the initial and present values of the energy of the gas, W is the kinetic energy of the material of the sphere, and E_Φ is the work of plastic deformation. We neglect the kinetic energy of the gas in the pore.

Let us consider each of the terms in Eq. (9) in more detail.

1. For an ideal gas the internal energy is $E = p_gV/(\gamma - 1)$, where V is the volume of the gas.

Using (6) and the fact that $V = (4/3)\pi a^3$, we obtain

$$E - E_0 = \left[4\pi a_0^3 p_{g0}/3(\gamma - 1)\right]\left[(a_0/a)^{3(\gamma-1)} - 1\right].$$

Since $\Phi_0 = (4/3)\pi a_0^3 p$,

$$\tilde{E} - \tilde{E}_0 = (E - E_0)/\Phi_0 = (p_{g0}/p(\gamma - 1))\,[(a_0/a)^{3(\gamma-1)} - 1]. \quad (10)$$

2. The kinetic energy of the sphere material is determined by the expression

$$W = \int_m (v^2/2)\, dm, \quad \text{where} \quad dm = 4\pi\rho r^2 dr.$$

Using Eq. (2), we obtain

$$W = 2\pi\rho a^3 \dot{a}^2 (1 - a/b),$$
$$\tilde{W} = W/\Phi_0 = (3\rho\dot{a}^2/2p)(1 - a/b)(a/a_0)^3. \quad (11)$$

3. The work of plastic deformation is

$$E_\Phi = \int_U A_p\, dU; \quad A_p = \int_0^{\varepsilon_\tau} \sigma_\tau\, d\varepsilon_\tau, \quad (12)$$

where σ_T and ε_T are, respectively, the stress and strain intensities, and U is the volume of the sphere.

Using the plasticity condition (3), and neglecting elastic deformations, we obtain

$$E_\Phi = 4\pi Y \int_a^b \varepsilon_\tau r^2 dr. \quad (13)$$

The tangential logarithmic strain is determined by the expression

$$\varepsilon_\theta = \ln(1 - u/r) = (1/3)\ln\left(1 - (a^3 - a_0^3)/r^3\right),$$

where u is the radial variable.

Using the fact that in the present case $\varepsilon_r = -2\varepsilon_\theta$, we obtain

$$\varepsilon_\tau = (2/3)\ln\left(1 - (a^3 - a_0^3)/r^3\right). \quad (14)$$

Substituting ε_T into Eq. (13) and integrating, we obtain

$$E_\Phi = -(8/3)\pi Y\left[a^3\ln(b/a) + b_0^3\ln(b/b_0) - a_0^3\ln(b/a_0)\right],$$
$$\tilde{E}_\Phi = E_\Phi/\Phi_0 = -2YC/p, \quad (15)$$

where $C = (a/a_0)^3 \ln(b/a) + (b_0/a_0)^3 \ln(b/b_0)^3 - \ln(b/a_0)$.

4. The work done by the external forces is given by the expression

$$\Phi = (4/3)\pi a^3 p$$

or

$$\tilde{\Phi} = \Phi/\Phi_0 = (a/a_0)^3. \quad (16)$$

Substituting (10), (11), (15), and (16) into Eq. (9), using Eq. (1), and making some transformations, we obtain

$$\dot{a} = -\left\{p\,\frac{1 - x^3 + p_{g0}[x^{3(1-\gamma)} - 1]/[p(\gamma - 1)] + 2YC/p}{(3/2)\rho x^3(1 - z)}\right\}^{1/2}, \quad (17)$$

where $C = -x^3\ln z + (1/m_0)\ln\left(x m_0^{1/3}/z\right) - \ln(x/z)$; x and z are determined from (8). The initial conditions are $a = a_0$ (x = 1) and $\dot{a} = 0$.

Using Eqs. (17) and (2) and the incompressibility condition (7), we can plot a curve for the velocity over the thickness of the sphere as a function of the position of the pore boundary.

By using Eqs. (8) and (17), the acceleration of the boundary can be determined:

$$\ddot{a} = \frac{p}{a_0\rho x(1 - z)}\left\{\frac{2}{3x^3}\left[\frac{(1 + z)(1 + z^2)}{2} - 2\right]\left[1 - x^3 + \frac{p_{g0}[x^{3(1-\gamma)} - 1]}{p(\gamma - 1)} + \frac{2YC}{p}\right] - \frac{2Y}{p}\ln z + \frac{p_{g0}}{p}x^{-3\gamma} - 1\right\}. \quad (18)$$

Values calculated from Eqs. (17) and (18) and those from a numerical solution of the differential equation (8) agreed within the accuracy of the numerical approximation.

The velocity of the pore boundary as a function of x is shown in Fig. 1. Calculations were performed for initial pressures of the gas in the pore $p_{g_0}/p = 0$ (curve 4), $p_{g_0}/p = 0.001$ (curves 1, 2, 5), and $p_{g_0}/p = 0.01$

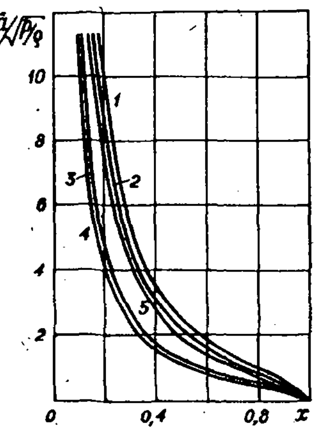

Fig. 1

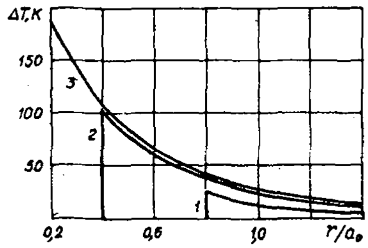

Fig. 2

(curve 3) for initial porosities 0.01 (curve 5) and 0.05 (curves 1-4). The polytropic exponent γ was taken equal to 1.4. The region of completely plastic flow of the layer is bounded by the value $Y/p_T \approx 0.326$ for $m_0 = 0.01$, and $Y/p_T = 0.501$ for $m_0 = 0.05$. The condition $Y/p = 0$ (curve 1) corresponds to pore collapse in an ideal incompressible liquid. The values $Y/p = 0.1$ (curves 2 and 5) and $Y/p = 0.3$ (curves 3 and 4) were used in the calculations.

The results obtained show that the initial pressure of the gas in the pore (curves 3 and 4) has no significant effect on the process of its deformation, and it is accurate enough to take $p_{g_0} = 0$ in performing calculations. The value of the initial porosity also has little effect on the pore deformation in high-density materials (curves 2 and 5). A change in m_0 by a factor of 5 leads to changes in the velocity of less than 10%.

To determine the change in temperature of the pore surface during its deformation we write the expression for the specific internal energy of the material in the form

$$E_T = A_p/\rho,$$

where A_p is determined from (12). Using the plasticity condition (3) and the relation for the strain intensity (14), we obtain

$$E_T = (2Y/3\rho)\ln(1-(a^3-a_0^3)/r^3). \tag{19}$$

The last equation does not take account of elastic deformations; i.e., it is assumed that pore collapse occurs in the plastic flow stage of the material.

Using Eq. (19), the temperature change over the thickness of the layer can be determined as a function of the position of the pore boundary:

$$\Delta T = E_T/c_p = (2Y/3\rho c_p)\ln[1-(r/a_0)^{-3}(x^3-1)], \tag{20}$$

where c_p is the specific heat at constant pressure, and r is the Eulerian coordinate determined from the incompressibility condition

$$r/a_0 = [(r_0/a_0)^3 + x^3 - 1]^{1/3}.$$

In particular, the temperature change at the bore boundary $r = a$ is

$$\Delta T = -(2Y/\rho c_p)\ln x. \tag{21}$$

Equations (20) and (21) determine the maximum heating of the pore surface during its collapse as a result of plastic deformation of the material.

We estimate the maximum heating by using the values $Y = 0.12$ GPa for the dynamic yield strength, and $m_0 = 0.05$ for the initial porosity found experimentally for the dynamic loading of TNT [7]. The density of the material ρ was taken equal to 1.66×10^3 kg/m^3, and the specific heat $c_r = 1.26 \times 10^3$ J/kg·deg. The curves in Fig. 2 show the temperature distribution over the thickness of the layer at various stages of the pore deformation process. The numbers 1-3 correspond to x = 0.8, 0.4, and 0.2 respectively. The results obtained show that only local heating of the material results from its plastic deformation, and does not exceed 200°K for the surface layers of a pore. The typical thickness of the heated spherical layer is $(0.70\text{-}0.85)a_0$. Here the increase in temperature of the surface of the pore as a result of adiabatic heating of the gas compressed in it is not taken into account, but this is quite admissible for pores up to several tens of microns in diameter because of the substantial difference between the coefficients of thermal activity of the gaseous and solid phases of the material [8].

The analysis has been limited to the special case of pore collapse in a plastically deforming material. The results obtained give a qualitatively correct description of the functional trends of the temperature distribution over the thickness of the layer. The quantitative results correspond to the phenomenology of the process, except for a spherical layer in the vicinity of the pore, where the effects of viscous flow play an important role.

LITERATURE CITED

1. V. V. Nikolaevskii, "Dynamic strength and rate of fracture," in: Shock, Explosion, and Fracture [in Russian], Mir, Moscow (1981).
2. W. Herrmann, "Constitutive equation for the dynamic compaction of ductile porous materials," J. Appl. Phys., 40, 2490 (1969).
3. M. A. Goodman and S. C. Cowin, "A continuum theory for granular materials," Arch. Rat. Mech. Anal., 44, 249 (1972).
4. M. M. Carroll and A. C. Holt, "Stark and dynamic pore-collapse relations for ductile porous materials," J. Appl. Phys., 43, 1626 (1972).
5. J. J. Bhatt, M. M. Carroll, and J. F. Schatz, "A spherical model calculation for volumetric response of porous rocks," Trans. ASME, Ser. E, J. Appl. Mech., 42, 363 (1975).
6. V. A. Odintsov and V. V. Selivanov, "Behavior of a rigid-plastic cylindrical shell under the action of external pressure," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 172 (1975).
7. R. J. Wasley and F. E. Walker, "Dynamic compressive behavior of a strain-rate sensitive polycrystalline organic solid," J. Appl. Phys., 40, 2639 (1969).
8. A. V. Dubovik and V. K. Bobolev, Sensitivity of Liquid Explosives to Shock [in Russian], Nauka, Moscow (1978).

CALCULATION OF AN INHOMOGENEOUS ELASTIC HALF SPACE AND A PLATE PLACED ON IT

G. P. Kovalenko

UDC 539.8

In many problems solved within the framework of the model of the linear theory of elasticity of isotropic media, it is necessary to take into account the variation of the properties of the medium as functions of the coordinates, particularly the coordinate z, the depth of a half space. Such problems arise in geophysics, seismology, and structural mechanics. Since the equations of motion of an elastic medium are interrelated, an effective analytic solution of boundary-value problems for any inhomogeneity is very difficult to obtain. However, as is shown in [1], the Lamé vector equation of motion permits separation into independent equations for an infinite set of inhomogeneous media. Assuming weak inhomogeneity of these media does not detract in practice from the generality of the results and, at the same time, permits effective solution of boundary-value problems by approximate methods. Real media can be approximated with sufficient accuracy of the results by media which belong to the above-mentioned set [2]. In the present study we obtain a solution for a problem in the vibrations of an infinite classical plate on an inhomogeneous half space when a vibrating load moves at constant velocity over the plate. We consider in more detail special cases of the problem which are obtained from the previous solution by a passage to the limit with respect to several parameters. When the medium becomes homogeneous, the functional relationships found become the results known for homogeneous media.

1. We consider an isotropic elastic half space in a Cartesian coordinate system with the positive direction of the axis OZ pointing downward. The equation of motion of the medium is taken in the form [1]

Sumy. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 132-140, January-February, 1983. Original article submitted December 29, 1981.